Graph Algorithms:

What are the elements of graphs?

* An abstract way of representing connectivity using nodes (also called vertices) and edges
* We will label the nodes from 1 to n
* m edges connect some pairs of nodes
* Edges can be either one-directional (directed) or bidirectional
* Nodes and edges can have some auxiliary information



What is a “digraph”?

* Also known as directed graph
* A digraph D is a pair (V, A), where V is a nonempty set whose elements are called the vertices and A is the subset of the set of ordered pairs of distinct elements of V.
* The concept of digraphs (or directed graphs) is one of the richest theories in graph theory, mainly because of their applications to physical problems. For example, flow networks with valves in the pipes and electrical networks are represented by digraphs. They are applied in abstract representations of computer programs and are invaluable tools in the study of sequential machines.

What is meant by a “weighted graph”?

* A [*graph*](https://xlinux.nist.gov/dads/HTML/graph.html) having a weight, or number, associated with each [*edge*](https://xlinux.nist.gov/dads/HTML/edge.html). Some algorithms require all weights to be nonnegative, integral, positive, etc.

Explain the following graph concepts:

Path

* path in a [graph](https://en.wikipedia.org/wiki/Graph_(discrete_mathematics)) is a finite or infinite [sequence](https://en.wikipedia.org/wiki/Sequence) of [edges](https://en.wikipedia.org/wiki/Edge_(graph_theory)) which connect a sequence of [vertices](https://en.wikipedia.org/wiki/Vertex_(graph_theory)) which, by most definitions, are all distinct from one another.

Cycle

* a cycle is a path of edges and vertices wherein a [vertex](https://en.wikipedia.org/wiki/Vertex_(graph_theory)) is reachable from itself.
* There are two types of cycles :
* closed walk : start and end at the same vertex and consecutive vertices are are adjacent to each other
* simple cycle: no repetition allowed

Subgraph

Bipartite

* A graph without odd cycles
* Nodes can be separated into two groups S
* and T such that edges exist between S and T only (no edges within S or within T)

Matching

* Set of edges without common edges / pairwise nonadjacent edges

Clique

* A completely interconnected set of edges and vertices which can be either a subgraph .

Independent set

* an independent set or stable set is a set of vertices in a graph, no two of which are adjacent. That is, it is a set S of vertices such that for every two vertices in S, there is no edge connecting the two.

What is a minimum spanning tree?

* A minimum spanning tree (MST) or minimum weight spanning tree is a subset of the edges of a connected, edge-weighted undirected graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight.

What is an Euler cycle?

* exists if and only if the graph is connected and the number of nodes with odd degree is 0 or 2.

What is a Hamiltonian cycle?

* a path/cycle that visits every node in the graph exactly once. Looks similar but very hard

Graph algorithms

Graph traversal: What is…

a “depth-first search”?

DFS(v): visits all the nodes reachable from v in depth-first order

◮ Mark v as visited

◮ For each edge v ->u:

–  If u  is not visited, call DFS(u )

◮ Use non-recursive version if recursion depth is too big (over a

few thousands)

–  Replace recursive calls with a stack

a “breadth-first search”?

BFS(v): visits all the nodes reachable from v in breadth-first order

◮ Initialize a queue Q

◮ Mark v as visited and push it to Q

◮ While Q is not empty:

–  Take the front element of Q  and call it w

–  For each edge w -> u :

◮ If u is not visited, mark it as visited and push it to Q

What problem is Kruskal’s algorithm for?

* To find a subset of E with the minimum total weight that connects all the nodes into a tree

What is the Travelling salesman problem?

* It’s the problem of finding a minimum weight Hamilton circuit in KN.